$$\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^2}\,dx$$ DISCONT @ $x=0$

$u = \frac{1}{x}$

$du = -\frac{1}{x^2}dx$

$\int -e^u du = -e^u$

$= -e^{\frac{1}{x}}$

$$= \lim_{N\to 0^-} \int_{-1}^{N} \frac{e^{\frac{1}{x}}}{x^2}\,dx$$

$$= \lim_{N\to 0^-} -e^{\frac{1}{x}}\Big|_{-1}^{N}$$

$$= \lim_{N\to 0^-} \left(-e^{\frac{1}{N}} + e^{-1}\right)$$

$\frac{1}{N} \to -\infty$

$$= e^{-1}$$

$$\int_0^\infty \frac{1}{x(\ln x)^3}\,dx$$ DISCONT @ $X=0, 1$

$U = \ln x$

$dU = \frac{1}{x}\,dx$

$$\int \frac{1}{U^3}\,dU = -\frac{1}{2U^2}$$

$$= -\frac{1}{2(\ln x)^2}$$

$$= \int_0^{\frac{1}{e}} \frac{1}{x(\ln x)^3}\,dx + \int_{\frac{1}{e}}^1 \frac{1}{x(\ln x)^3}\,dx$$

$$+ \int_1^e \frac{1}{x(\ln x)^3}\,dx + \int_e^\infty \frac{1}{x(\ln x)^3}\,dx$$

$$= \lim_{N\to 1^+} \int_N^e \frac{1}{x(\ln x)^3}\,dx$$

$$= \lim_{N\to 1^+} -\frac{1}{2(\ln x)^2}\Big|_N^e$$

$$= \lim_{N\to 1^+} \left(-\frac{1}{2(\ln N)^2} + \frac{1}{2}\right)$$

$\ln N \to 0$

$$= -\infty$$

DIVERGES

$$\int \frac{8x-48}{x^4-16}\,dx$$ **HINT: Do NOT factor the numerator**

$$\frac{8x-48}{(x+2)(x-2)(x^2+4)} = \frac{A}{x+2} + \frac{B}{x-2} + \frac{C(2x)+D(2)}{x^2+4}$$

$$8x-48 = A(x-2)(x^2+4) + B(x+2)(x^2+4) + C(2x)(x+2)(x-2) + D(2)(x+2)(x-2)$$

$x=2$: $-32 = B(4)(8) \rightarrow B=-1$

$x=-2$: $-64 = A(-4)(8) \rightarrow A=2$

$x=0$: $-48 = A(-2)(4) + B(2)(4) + D(2)(2)(-2)$

$-48 = -16-8-8D \rightarrow D=3$

COEF OF x^3: $0 = A+B+2C$

$0 = 2-1+2C \rightarrow C = -\frac{1}{2}$

SANITY CHECK: $x=3$

LHS $= \frac{-24}{(5)(1)(13)} = \frac{-24}{65}$

RHS $= \frac{2}{5} - \frac{1}{1} + \frac{-\frac{1}{2}(6)+3(2)}{13}$

$= -\frac{3}{5} + \frac{3}{13}$

$= \frac{-39+15}{65}$

$= \frac{-24}{65}$ ✓

$$\int \left(\frac{2}{x+2} - \frac{1}{x-2} + \frac{-\frac{1}{2}(2x)+3(2)}{x^2+4}\right)dx$$

$$= 2\ln|x+2| - \ln|x-2| - \tfrac{1}{2}\ln(x^2+4) + 3\tan^{-1}\tfrac{x}{2} + C$$

$$\int \sec^2 \sqrt{x}\, dx$$

$$u = \sqrt{x}$$
$$x = u^2$$
$$dx = 2u\, du$$

$$= \int 2u \sec^2 u \, du$$

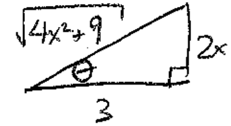
$$\begin{array}{lcl} 2u & + & \sec^2 u \\ 2 & - & \tan u \\ 0 & & -\ln|\cos u| \end{array}$$

$$= 2u \tan u + 2 \ln|\cos u| + C$$

$$= 2\sqrt{x} \tan \sqrt{x} + 2 \ln|\cos \sqrt{x}| + C$$

Find $\int x^2\sqrt{4x^2+9}\,dx$. **SCORE: _____ / 30 PTS**

$$x=\frac{3}{2}\tan\theta \longrightarrow \tan\theta=\frac{2x}{3}$$



$$dx=\frac{3}{2}\sec^2\theta\,d\theta$$

$$\int\left(\frac{9}{4}\tan^2\theta\right)(3\sec\theta)\left(\frac{3}{2}\sec^2\theta\right)d\theta$$

$$=\frac{81}{8}\int\tan^2\theta\sec^3\theta\,d\theta$$

$$=\frac{81}{8}\int(\sec^5\theta-\sec^3\theta)\,d\theta$$

$$=\frac{81}{8}\left[\frac{1}{4}\sec^3\theta\tan\theta+\frac{3}{4}\int\sec^3\theta\,d\theta-\int\sec^3\theta\,d\theta\right]$$

$$=\frac{81}{8}\left[\frac{1}{4}\sec^3\theta\tan\theta-\frac{1}{4}\left(\frac{1}{2}\sec\theta\tan\theta+\frac{1}{2}\ln|\sec\theta+\tan\theta|\right)\right]+C$$

$$=\frac{81}{32}\sec^3\theta\tan\theta-\frac{81}{64}\sec\theta\tan\theta-\frac{81}{64}\ln|\sec\theta+\tan\theta|+C$$

$$=\frac{81}{32}\left(\frac{\sqrt{4x^2+9}}{3}\right)^3\left(\frac{2x}{3}\right)-\frac{81}{64}\left(\frac{\sqrt{4x^2+9}}{3}\right)\left(\frac{2x}{3}\right)-\frac{81}{64}\ln\left|\frac{\sqrt{4x^2+9}}{3}+\frac{2x}{3}\right|+C$$

$$=\frac{1}{16}x\left(\sqrt{4x^2+9}\right)^3-\frac{9}{32}x\sqrt{4x^2+9}-\frac{81}{64}\ln\left|\sqrt{4x^2+9}+2x\right|+C$$

Determine if $\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$ converges or diverges. Justify your answer properly. **SCORE: _____ / 15 PTS**

$$0 \leq \frac{1}{x\sqrt{x}} \leq \frac{\sec^2 x}{x\sqrt{x}}$$

$$\hookrightarrow \int_0^1 \frac{1}{x^{\frac{3}{2}}}\,dx \text{ DIVERGES } \left(p = \tfrac{3}{2} > 1\right)$$

$$\text{SO } \int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx \text{ DIVERGES}$$